Economics without Borders

Economic Research for European Policy Challenges

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Big Data in Economics: Evolution or Revolution?

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Abstract

The Big Data Era creates a lot of exciting opportunities for new developments in economics and econometrics. At the same time, however, the analysis of large data sets poses difficult methodological problems which should be addressed in an appropriate way and are the subject of the present chapter.

14.1 Introduction

'Big Data' has become a buzzword both in academic circles and in business and policy circles. It is used to cover a variety of data-driven phenomena that have very different implications for empirical methods. This chapter discusses some of these methodological challenges.¹

In the simplest case, 'Big Data' means a large data set that has otherwise a standard structure. For example, Chapter 13 describes how researchers are increasingly gaining access to administrative data sets or business records covering entire populations rather than population samples. The size of these data sets allows for better controls and more precise estimates and is a bonus for researchers. This may raise

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challenges for data storage and handling, but it does not raise any distinct methodological issues.

However, 'Big Data' often means much more than just large versions of standard datasets. First, large numbers of units of observation often come with large numbers of variables. That is large numbers of possible covariates. To illustrate with the same example, the possibility to link different administrative data sets increases the number of variables attached to each statistical unit. Likewise, business records typically contain all interactions of the customers with the business. This can create a tension in the estimation between the objective of 'letting the data speak' and obtaining accurate (in a way we will make precise) coefficient estimates. Second, Big Data sets often have a very different structure from the those we are used to in economics. This includes web search queries, real-time geo-locational data or social media to name a few. This type of data raises questions about how to structure and possibly re-aggregate them.

The chapter starts with a description of the 'curse of dimensionality' which arises from the fact that both the number of units of observations and the number of variables associated with each unit are large. This feature is present in many of the Big Data applications of interest to economists. One extreme example of this problem occurs when there are more parameters to estimate than observations. In this case standard estimators (such as ordinary least squares) do not yield a unique solution. The section, which borrows heavily from De Mol et al. (2008), describes the econometric problems raised by the curse of dimensionality. It describes some of the methodological solutions, called regularisation methods, that have been proposed.

Section 14.3 then discusses recent research on recovering policy effects using Big Data. In many fields in economics, we are interested in measuring a (causal) relationship between some variable of interest (for example, a policy) and its effects. In other words, although there might be many variables, some of them (related to a specific policy) are of special interest to the researcher. The section describes current efforts to develop methods that combine the ability of regularisation methods to harness the information contained in these richer data sets, with the possibility to identify the impact of specific policy relevant effects.

Section 14.4 turns to prediction problems. In prediction problems, we are not interested in specific coefficients per se but in our ability to forecast a variable of interest, *e.g.*, inflation, growth or the probability of default. Forecasting has a long tradition in macroeconomics and the greater availability of highly granular microdata is renewing interest in prediction problems at the microeconomic level too. A priori, regularisation methods are well-suited for this type of problem. However, 'off-the-shelf' regularisation methods are agnostic regarding the data generation process. The section argues, on the basis of the experience with macro forecasting

models, for the need to develop regularisation methods that account for the specificities of the data generation processes in economics, such as serial correlation or mixed frequencies.

Recent progress in computing power and storage capacities has allowed researchers to handle and analyse increasingly big data sets. For some of the Big Data (*e.g.*, high frequency trading data, browsing data), this may not be enough. Section 14.5 discusses how simulation-based methods can be refined to leverage the potential of parallel computing.

Section 14.6 concludes. The availability of unprecedented amounts of data offers exciting research opportunities in economics. While researchers will be able to exploit some of the methods developed in other fields, such as statistics and computer science, it is nevertheless essential that some of these methods be tailored to the specificities of economic research questions and economic data. On these fronts, there is still much to be done.

14.2 The Curse of Dimensionality and Regularisation

An early occurrence of the term 'Big Data' in economics is to be found in a discussion by Diebold (2003, 2012). To quote "I stumbled on the term Big Data innocently enough, via discussion of two papers that took a new approach to macroeconometric dynamic factor models (DFMs), Reichlin (2003) and Watson (2003), presented back-to-back in an invited session of the 2000 World Congress of the Econometric Society."

The two authors referenced above were presenting their research on factor models in high-dimensional time series (Forni et al. (2000); Stock and Watson (2002)), which mainly consisted in deriving asymptotic results for the case where both the number of time samples and the cross-sectional dimension, *i.e.*, the number of time series, tend to infinity. The approach relied on a factor model dating back to Chamberlain and Rothschild (1983) in finance, but generalised to take serial correlation into account. Stock and Watson (2002) considered so-called 'static' factor models whereas Forni et al. (2000) derived asymptotics in the case of 'dynamic' factor models. The estimators they used are based on a few principal components either in the time domain for the static case or in the Fourier domain for the dynamic case. This factor-model literature was probably the first in economics to address the difficulties arising from the high dimensionality of the data, albeit under rather strong assumptions (namely factor models) on the data generating process.

In statistics, the difficulties pertaining to the analysis of high-dimensional data are well-known issues, often referred to as the 'curse of dimensionality'. Some of the facets of this curse can be explained using the familiar example of the linear re-

gression model. To introduce some background notation useful for our discussion, let

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{U} \tag{14.1}$$

where **X** is a $n \times p$ matrix containing the observed predictors (covariates), **Y** is the outcome or $n \times 1$ vector of the observed responses and **U** is an unobserved zero-mean error or nuisance term. The $p \times 1$ vector β contains the regression coefficients. In the case of time series, n is the number of time samples and p is the number of observations and p is the number of covariates. In the discussion in this section, we will consider the matrix **X** as deterministic.

Depending on the application under study, two different problems can be highlighted. The first one is prediction (also referred to as 'generalisation' by the machine-learning community), in which case one is only interested in estimating the outcome for future times or new examples to come. This requires the estimation of the regression parameters, but only as an auxiliary step to the estimation of the outcome. The second problem, the identification of the model, pertains more to the vector β of regression coefficients itself, in the linear regression example in (14.1). This is essential for interpreting the estimated coefficients in terms of their relevance in predicting the response. For example, some coefficients can be zero, indicating that the corresponding predictors are not relevant for this task. The determination of these zeroes, hence of the relevant/irrelevant predictors, is usually referred to as 'variable selection'.

As is well known, the most straightforward solution for the linear regression problem is Ordinary Least Squares (OLS). The OLS estimator for β in (14.1) minimises the least-squares loss

$$\Phi(\beta) = \|\mathbf{Y} - \mathbf{X}\beta\|_2^2 , \qquad (14.2)$$

where $\|\mathbf{Y}\|_2 = \sqrt{\sum_{i=1}^n |Y_i|^2}$ is the L_2 -norm of the vector \mathbf{Y} . It is given by

$$\hat{\beta}_{ols} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} \tag{14.3}$$

(X') denotes the transpose of the matrix X).

For the OLS estimator, expression (14.3), to make sense we need the $p \times p$ matrix $\mathbf{X}'\mathbf{X}$ to be of full rank, hence invertible. This cannot be the case in high-dimensional situations where the number of coefficients, p, is larger than the number of observations, n. ² In that case, the minimiser of the least-squares loss is non-unique, but uniqueness can be restored by selecting the so-called 'minimum-norm least-squares solution', orthogonal to the null-space, *i.e.*, by ignoring the subspace corresponding to the zero eigenvalues.

Since this implies the existence of a non-trivial null-space for X'X, with at least p-n zero eigenvalues.

Notice that although this remedy may work well for prediction, the identification problem remains hindered by this non-uniqueness issue. An additional difficulty arises when the matrix X'X has eigenvalues that are close to zero, or, more precisely, when its 'condition number', i.e., the ratio between the largest and the smallest of its nonzero eigenvalues, becomes large. Such situation prevents a stable determination of the least-squares (or minimum-norm least-squares) estimator: small fluctuations in the outcome vector Y will be amplified by the effect of the small eigenvalues and will result in large uncontrolled fluctuations (high variance/volatility) on the estimation of β , again preventing meaningful identification.

The pathologies described above contribute to what is often referred to as the 'curse of dimensionality' or else the 'large p, small n paradigm' in high-dimensional statistics. As early as in the fifties, Stein (1956) introduced a 'high-dimensional' surprise in statistics by showing that the maximum-likelihood estimator of the unknown mean vector of a multivariate Gaussian distribution is not "admissible" in dimension higher than three, i.e., that it is outperformed by 'shrinkage' estimators. Heuristically, 'shrinking' means that a naive estimate is improved by combining it with other information or priors.

Many remedies have been proposed to address these pathologies, under the common appellation of 'regularisation methods', which provide in one form or another the dimensionality reduction which is necessary to reduce the variance/volatility of the unstable estimators or in other words to avoid 'overfitting'. Overfitting refers to the fact that, when using a model with many free parameters (here the p components of β), it is easy to get a good fit of the observed data, i.e., a small value for the residual (14.2), but that this does not imply that the corresponding (unstable) value of β will have a good predictive power for responses corresponding to new observations. For time series, good in-sample fit does not imply good out-of-sample forecasts.

Box 1: Principal Component Regression (PCR)

The Principal Component Regression consists in estimating β by

$$\hat{\beta}_{pcr} = \sum_{i=1}^{k} \frac{\langle \mathbf{X}' \mathbf{Y}, \mathbf{V}_i \rangle}{\xi_i^2} \, \mathbf{V}_i \tag{14.4}$$

where the V_i 's are the eigenvectors of X'X with eigenvalues ξ_i^2 , and $\langle \cdot, \cdot \rangle$ denotes the scalar product.

One of the simplest regularisation methods is principal component regression (PCR), a statistical procedure that transforms the possibly correlated variables into a smaller number of orthogonal new variables (the components), see Box 1. The truncation point k for the number of components, usually much smaller than the true rank of **X'X**, has to be carefully chosen to overcome the instabilities. In this method, also referred to as '*Truncated Singular Value Decomposition*' (TSVD), the truncation introduces a bias in order to reduce the variance.

Until recently alternative estimators were less well known in econometrics. Other regularisation methods introduce constraints or penalties on the vector $\boldsymbol{\beta}$ of the regression coefficients. Probably the oldest penalised regression method is 'Ridge regression', see Box 2, due to Hoerl and Kennard (1970). This method is also known in the applied mathematics literature as Tikhonov's regularisation. It consists in adding to the least-squares loss a penalty proportional to the size of $\boldsymbol{\beta}$, measured by its squared L_2 -norm. As for the truncation point in PCR, the regularisation parameter has to be chosen carefully in order to provide a proper balance between the bias introduced by shrinkage and the variance of the estimator and its value is usually determined by cross-validation.

Box 2: The Ridge Regression Estimator

The ridge regression estimator is given by

$$\hat{\beta}_{ridge} = \operatorname{argmin}_{\beta} \left[\|\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}\|_{2}^{2} + \lambda_{2} \|\boldsymbol{\beta}\|_{2}^{2} \right]
= (\mathbf{X}'\mathbf{X} + \lambda_{2}\mathbf{I})^{-1}\mathbf{X}'\mathbf{Y}$$
(14.5)

where **I** is the identity matrix and $\lambda_2 > 0$ is the so-called '*regularisation* parameter', which, as seen from (14.5), reduces the impact the smallest eigenvalues of **X**'**X**, at the origin of the instability of the OLS estimator.

Ridge regression introduces a form of linear 'shrinkage', where the components of $\hat{\beta}_{ols}$ are shrunk uniformly towards zero, as can be easily seen in the case of orthonormal regressors (i.e., for $\mathbf{X}'\mathbf{X} = \mathbf{I}$), where $\hat{\beta}_{ridge} = \frac{1}{1+\lambda_2} \mathbf{X}'\mathbf{Y}$. More generally, quadratic penalties provide estimators which depend linearly on the response \mathbf{Y} but do not allow for variable selection, since typically all regression coefficients are different from zero.

An alternative to quadratic penalties that allows for variable selection by enforcing sparsity, *i.e.*, the presence of zeroes in the vector β of the regression coefficients, has been popularised in the statistics and machine-learning literature by Tibshirani (1996) under the name of 'Lasso regression'. It consists in replacing the L_2 -norm penalty used in ridge regression by a penalty proportional to the L_1 -norm of β , see Box 3.

Hence, the Lasso penalty provides a nonlinear shrinkage of the components of $\hat{\beta}_{ols}$, which are shrunk differently according to their magnitude, as well as sparsity, since the jth coefficient $[\hat{\beta}_{lasso}]_j = 0$ if $|[\mathbf{X'Y}]_j| < \lambda_1/2$. Unfortunately, there is no closed-form expression for $\hat{\beta}_{lasso}$ in the case of general matrices \mathbf{X} and the Lasso

estimator has to be computed numerically as the solution of a (nonsmooth) convex optimisation problem.

Box 3: The Lasso Regression Estimator

Lasso consists in replacing the L_2 -norm penalty used in ridge regression by a penalty proportional to the L_1 -norm of β , *i.e.* to the sum of the absolute values of the regression coefficients, $\|\beta\|_1 = \sum_{j=1}^p |\beta_j|$, yielding the estimator

$$\hat{\beta}_{lasso} = \operatorname{argmin}_{\beta} \left[\|\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}\|_{2}^{2} + \lambda_{1} \|\boldsymbol{\beta}\|_{1} \right]. \tag{14.6}$$

In the special case of orthonormal regressors (X'X = I), the Lasso estimator is easily seen to be given by

$$[\hat{\beta}_{lasso}]_j = S_{\lambda_1}([\mathbf{X}'\mathbf{Y}]_j)$$

where $S_{\lambda_1}(x)$ is the 'soft-thresholder' defined by

$$S_{\lambda_1}(x) = \begin{cases} x + \lambda_1/2 & \text{if} \quad x \le -\lambda_1/2 \\ 0 & \text{if} \quad |x| < \lambda_1/2 \\ x - \lambda_1/2 & \text{if} \quad x \ge \lambda_1/2 \end{cases}.$$

The previous estimators can be given a Bayesian interpretation, since $\hat{\beta}_{ols}$ can be viewed as the maximum (log-)likelihood estimator for a Gaussian error term and the penalized maximum likelihood estimators $\hat{\beta}_{ridge}$ and $\hat{\beta}_{lasso}$ can be interpreted as maximum a posteriori (MAP) estimators, the penalty resulting from a prior distribution for the regression coefficients. In Ridge regression, it corresponds to a Gaussian prior whereas in Lasso regression it is a Laplacian or double-exponential prior.

The regularisation techniques described above are paradigmatic since they convey the essential ideas in dealing with high-dimensional settings. There are however numerous extensions and generalisations. For example, more general types of penalties can be used such as $\|\beta\|_{\gamma}^{\gamma} = \sum_{j=1}^{p} |\beta_{j}|^{\gamma}$, i.e., the L_{γ} -norms used in 'bridge regression' (Frank and Friedman (1993)). Notice that in this family, though, only the choice $\gamma=1$ yields both convexity and sparsity. Moreover, weights or even a non-diagonal coupling matrix can be introduced in the penalty to cover the case of non i.i.d. (independent and identically distributed) regression coefficients. Composite penalties are also used e.g., in elastic-net or group-lasso regularisation. Finally, different loss functions can be considered such as those used in robust statistics, logistic regression, etc. A good pointer to this variety of techniques is the book by Hastie et al. (2009).

Let us remark that global variable selection methods, preferably convex to fa-

cilitate computation, such as the Lasso and its relatives, are essential to deal with high-dimensional situations. Indeed, considering all possible submodels and selecting the best among them, for example according to the Akaike Information Criterion (AIC) proposed by Akaike (1974) or the Bayesian Information Criterion (BIC) proposed by Schwarz (1978), leads to a complexity growing exponentially with the number of variables involved and renders the methods totally unpractical. To paraphrase the title of a paper by Sala-I-Martin (1997): "You cannot just run two million regressions!" (and, incidentally, two million would not even suffice for $p \geq 22$).

As concerns asymptotic and consistency results, the settings have to go beyond the classical scheme of keeping the number of parameters p constant (and usually small) while letting the number of observations n of the dependent variable tend to infinity. In high-dimensional situations, both n and p may tend to infinity, while assuming or not some relationship between their respective growth rates. The theory is more subtle in this case and is still developing. This question has been studied for principal component regression for time series under a factor model assumption. Results in this line for the case of penalised regression, and in particular of Ridge regression, have been derived by De Mol et al. (2008, 2015). The first paper also contains an empirical part where predictive accuracy of PCR, Ridge and Lasso regression is evaluated based on a data set of about 100 time series. It is shown that all three methods perform similarly and that results of Lasso are uninformative when used for applications where, which is typically the case for macroeconomics, data are cross-correlated. Moreover, in that case Lasso is unstable in selection.

14.3 Policy Analysis and Causal Inference

In the actual big data activity sphere, in parallel with the developments of powerful machine-learning techniques, the emphasis is on predictive rather than on causal models. As we shall further discuss in the next section, successful predictive algorithms are rapidly developing in response to the increasing demand coming from all kinds of applications. These algorithms convert large amount of unstructured data into predictive scores in an automatic way and often in real time.

Whether this trend is or not desirable may be a matter of debate but it is clear that it implies a significant shift from the single-covariate causal-effect framework that has dominated much empirical research especially in microeconomics. Being non-structural, predictive models are subject to the Lucas critique (Lucas (1976)) and their success should not obscure the fact that many economic applications are about inference on a causal effect. In microeconomics, for example, a successful

literature has developed methods to assess the effectiveness of a given policy or treatment.

In the case where the intervention is binary in nature, we define a binary variable, W, equal to unity for the treatment group and zero for the control group. One typically has in mind a counterfactual setting, where it makes sense to think of potential outcomes for each unit in the control and treated states. These outcomes are often denoted Y(0) and Y(1), and then we observe the treatment status, W, and the outcome under the corresponding treatment status, Y = (1 - W)Y(0) + WY(1) = Y(0) + W[Y(1) - Y(0)]. For unit i in the population, the treatment effect is $Y_i(1) - Y_i(0)$ — which is not observed. Instead, attention typically centers on the average treatment effect, $\tau = E[Y(1) - Y(0)]$, or the average over an interesting subpopulation (such as those actually receiving the treatment). A special case is when the treatment effect is constant, in which case we can write $Y = \tau W + Y(0)$, and the Y(0) plays the role of the unobserved factors affecting Y.

The potential outcomes setting can be extended to cases where the policy variable, W, is not binary. If the policy effect is constant across units and across levels of the treatment, we can write a simple regression equation

$$Y = \tau W + R, \tag{14.7}$$

where Y, W and R are random variables and τ is a scalar coefficient of interest. We (eventually) observe data on Y and W. The variable R includes unobserved factors -Y(0) in the simplest setting - affecting Y.

In medicine and the experimental sciences, truly randomised experiments can be carried out, which means the treatment level W can be made independent of R. For example, when W is binary, we can randomly assign individuals into the treatment and control groups. In such cases, (14.7) can be estimated using simple regression, which delivers an unbiased and consistent estimator of τ . In economics, random assignment is much less common, and in general one has only access to so-called observational – not experimental – data. Hence, several strategies when randomised assignment is not available have been developed. Here we review a few of those strategies, highlighting how high-dimensional regression methods can be applied to estimating causal effects. Good pointers to part of the relevant work in this field are the review papers by Belloni et al. (2013, 2014a).

A traditional and still commonly used method to handle nonrandom treatment assignment is regression adjustment, where one assumes the availability of covariates that render the policy assignment appropriately "exogenous." Let X be a $1 \times p$ vector of covariates. Then, if X is thought to predict both Y and the treatment assignment, W, we can "control for" X in a multiple regression analysis. These leads

to a linear model,

$$Y = \tau W + X\beta + U,\tag{14.8}$$

where now $R = X\beta + U$ and, if the elements of X suitably control for the nonrandom assignment, the treatment and covariates satisfy the exogeneity conditions

$$E[WU] = 0, E[XU] = 0.$$
 (14.9)

If p, the number of control variables, is large and the $p \times 1$ vector β is sparse, the model can be estimated by means of a Lasso regression, as described in the previous section. However, one has to know in advance the right vector X such that, under the usual exogeneity conditions (14.9), there are no more confounding variables and one recovers the marginal effect τ , holding fixed everything else.

One can relax the linearity assumption in X and just assume

$$E[R|W,X] = E[R|X],$$
 (14.10)

which yields

$$Y = \tau W + g(X) + U, \tag{14.11}$$

where g(X) = E[R|X] and U = R - E[R|W,X] is such that E[U|W,X] = 0. Model (14.11) is a so-called partially linear model and g is generally a nonparametric function. Belloni et al. (2014b) use Lasso-type methods to nonparametrically partial out X from both Y and W. They approximate the mean functions E[Y|X] and E[W|X] using functions of the form $\sum_{j=1}^{p} \beta_j \phi_j(X)$, for a large dictionary of functions $(\phi_j)_{j=1}^p$, and build a confidence interval around τ . This method is particularly appealing as it does not require one to chose a bandwidth to estimate the nonparametric conditional mean functions. If the approximations $\sum_{j=1}^{p} \beta_j \phi_j(X)$ are sparse, then the method selects the significant $\phi_i(X)$ for each of E[Y|X] and E[W|X]. As shown in Belloni et al. (2014a), using the union of the functions selected from the methods in a standard regression analysis with Y as the response variable and W as the other regressor, the usual heteroscedasticity-robust standard error produces valid t statistics and confidence intervals. It should be emphasised that, while the approach works well for selecting functions of X that appear in the conditional mean, it does not select the variables X such that (14.10) holds; the researcher is assumed to have already selected the appropriate controls.

When (14.9) or (14.10) do not hold, we can rely on instrumental variables, namely, assume to have at our disposal a vector of random variables Z, called instrumental variables, such that in (14.7),

$$Cov[Z, R] = 0.$$
 (14.12)

This yields the relation

$$Cov[Z, Y] = \tau Cov[Z, W]. \tag{14.13}$$

If Z is a scalar, (14.13) identifies τ when $Cov[Z,W] \neq 0$. When we have more than one instrumental variable for W, two stage least squares (2SLS) is a common estimation approach. However, 2SLS only uses the linear projection of W on Z in forming instruments. If we strengthen the exogeneity requirement to E[R|Z] = 0, 2SLS is asymptotically inefficient if E[W|Z] is nonlinear or Var[R|Z] is not constant. If we assume homoscedasticity in (14.7), that is, Var[R|Z] = Var[R], then the optimal instrument for W is given by E[W|Z], the best mean square predictor of W. Belloni et al. (2012) propose to use Lasso-type methods to estimate the regression function E[W|Z] using a large dictionary of approximating functions, and they show how to conduct inference on τ using a heteroscedastic-robust standard errors.

Gautier and Tsybakov (2011) propose an instrumental variables method to make inference for high-dimensional structural equations of the form

$$Y = X\beta + U \tag{14.14}$$

where the dimension of X is large (and may include exogenous and endogenous variables). This occurs, for example, in large demand systems, or when treatment variables are interacted with (exogenous) group dummies. In the latter case, the policy might have an effect on only certain groups and the policy maker would like to determine for which group the policy has an effect. The instrumental variables literature has identified various problems: (i) the instrumental variable candidates, Z, might not be exogenous; (ii) the instrumental variables can be 'weak' and estimating in a first-stage a reduced form equation can yield multimodal and nonnormal distributions of the parameter estimates, even with very large sample size, so that asymptotic theory is not reliable; (iii) in the presence of many instrumental variables, estimating in a first-stage a reduced form equation can give rise to a large bias. Gautier and Tsybakov (2011) rely on a new method which is robust to (ii) and (iii) in order to treat the more challenging case of a high-dimensional structural equation. Confidence sets can be obtained for arbitrary weak and numerous instrumental variables, whether or not the condition Cov[Z, U] = 0 gives rise to a unique β . Therefore, it is also possible to handle the case where the dimension of Z is smaller than the dimension of X, which can yield identification of β under sparsity of the structural equation (14.14) or other shape restrictions. To deal with possibility of (i), a high-dimensional extension of the Sargan and Hansen method is developed.

There is much interest in the treatment effects literature on heterogeneous treatment effects, and variable selection methods can also be applied in such cases. For

example, variable selection methods can be applied to estimating the propensity score when the treatment variable takes on a small number of levels. Moreover, variable selection methods can be applied to both propensity score estimation and regression function estimation to obtain so-called *doubly robust* estimators. Unlike the linear, additive equation (14.11), methods that weigh by the inverse of the propensity score allow for heterogeneous treatment effects. See, for example, the papers by Farrell (2015) and Athey and Imbens (2015).

Besides these high-dimensional problems, let us mention another important issue which arises in connection with the availability of big data sets, namely, to determine whether the accumulation of data, say, over an entire population, affects precision of estimates. Abadie et al. (2014) analyse how to compute uncertainty in empirical situations where the sample is the entire population and where the regression function is intended to capture causal effects. Other contributions on *Causal Inference* in a big data setting use machine-learning methods. A recent example is the work by Athey and Imbens (2015). There are many open challenges in this area and pointers to recent progress are available from the site of the Sackler Colloquium on "Drawing Causal Inference from Big Data", organised in March 2015 at the US National Academy of Science in Washington.³

The previous discussion has focused on cross-sectional data, but empirical researchers attempting to estimate causal effects often rely on panel data that exploit changes in policies over time. An important component of panel data models is allowing for time-constant, unobserved heterogeneity. Belloni et al. (2014a) propose first differencing a linear unobserved effects equation to remove additive heterogeneity, and then using variable selection methods, such as Lasso, to allow for correlation between unobserved heterogeneous trends and unknown functions of observed covariates – including the policy variable or variables being studied. The approach seems promising. So far, such methods have been applied to linear models with relatively few sources of heterogeneity.

14.4 Prediction

Despite the recent advances in identification and causality in big data settings that we have just reviewed, it is fair to say that the literature in the field is mainly focused on prediction. Using the same notation as above, the problem consists in computing the conditional expectation

$$E(Y|W,X). (14.15)$$

³ See http://www.nasonline.org/programs/sackler-colloquia/completed_colloquia/Big-data.html

Forecasting has a long tradition in economics, especially in macroeconomics. Indeed many economists in the private sector and policy institutions are employed to this task. In forecasting, robustness is typically tested in out-of-sample validation studies, a perspective typically ignored in empirical microeconomics. For desirable out-of-sample performance, models must respect the principle of parsimony (*i.e.*, contain a rather small number of free parameters) to avoid overfitting. However, the curse of dimensionality problem naturally arises from lags, non-linearities, and the presence of many potentially relevant predictors.

The recent literature has suggested methods to deal with the curse of dimensionality issue in dynamic models. Here we should mention dynamic factor models cited earlier and, more recently, large Bayesian vector autoregressive models. Following the work of De Mol et al. (2008), Banbura et al. (2010) have shown empirically how to set priors to estimate a vector autoregressive model with large data sets. The idea there is to set the degree of 'shrinkage' in relation to the dimension of the data. Intuitively this implies to set priors so as to avoid overfitting, but still let the data be informative. Giannone et al. (2015) have developed a formal procedure to conduct inference for the degree of shrinkage. These models have many applications in economics beyond pure forecasting and can be used to design counterfactuals for policy analysis and identification of exogenous shocks and dynamic propagation mechanisms. Large data allow to better identify exogenous shocks since it can accommodate for realistic assumptions on agents' information set (for an analysis on this point, see Forni et al. (2009)).

One very successful application of the large models described above (if measured by impact on modelling in policy institutions and the financial industry) has been 'now-casting'. Now-casting is the forecast of the present (present quarter or present month) based on data available at different frequencies (daily, weekly, monthly and quarterly). A now-cast produces a sequence of updates of predictions in relation to the publication calendar of the data. This allows to exploit the timeliness of certain data releases to obtain an early estimate of those series which are published with a delay with respect to the reference quarter such as GDP or the reference month such as employment (see Giannone et al. (2008) and subsequent literature). Empirical results show that exploiting survey data, which are published earlier than hard data, allows to obtain an accurate early estimate at the beginning of the quarter and, as new data are released through time, the estimates become more accurate (see Banbura et al. (2013) for a review of the literature). In principle, non-standard data such as Google queries or twitters, due to their timeliness, could be exploited in this context. However, once the details of the problem (mixed frequency, non-synchronous data releases) are appropriately modelled and relevant timely indicators considered, there is no evidence that Google indexes used successfully in simpler setup by Choi and Varian (2012) and Scott and Varian (2014) have any additional predictive value Li and Reichlin (2015), but more research is needed on this topic.

It has to be noted that most of the applied work on the methods mentioned have concerned traditional time series (macroeconomic variables, possibly disaggregated by sectors or regions, financial variables and surveys) and rarely with dimension above 150. Empirical results show that, in general, forecasts of macroeconomic variables based on data sets of medium dimension (of the order of 20) are not outperformed by forecasts based on 100 or more variables although the dimension helps especially in now-casting where successful results rely on the use of timely information. Moreover, as mentioned in Section 14.2, Lasso regressions provide unstable variable selection due to the near-collinear feature of macroeconomic data. Important empirical issues are also related to robustness with respect to variable transformation such as de-seasonalisation or detrending as well as nonlinearity. Potentially, machine-learning type of techniques could be useful in this set-up but this is open to future research. The literature is too much at an early stage to provide a definitive answer on the potentials of new data and new methods in this context but it is our view that any successful applications have to incorporate the detailed micro-structure of the problem. In now-casting, for example, this implies taking care of mixed frequency, non-synchronicity of releases and other details.

In microeconometrics the emphasis on predictions and out-of-sample is newer than in macro but studies using big data are more numerous. Predictions based on large cross-section of data have been successfully obtained for various problems. Examples can be found in the papers by Varian (2014), by Einav and Levin (2014) and by Kleinberg et al. (2015b), as well as in the references therein. The last paper discusses a problem of health economics, namely the prediction of whether replacement surgery for patients with osteoarthritis will be beneficial for a given patient or not, based on more than 3000 variables recorded for about 100 000 patients. Another policy decision based on prediction is studied by Kleinberg et al. (2015a) who show that machine-learning algorithms can be more efficient than a judge in deciding who has to be released or go to jail while waiting for trial because of danger of committing a crime in the meanwhile. Another application would be to predict the risk of unemployment for a given individual based on a detailed personal profile.

It should be remarked that machine-learning algorithms present several advantages: they focus on a best-fit function for prediction, possibly handling very rich functional forms, and have built-in safeguards against overfitting so that they can handle more variables than data points. Moreover, they do not require too many assumptions about the data generating process as it is the case in classical econometrics. One should be aware, however, that precisely because of their great generality

and versatility, they may not be optimally tailored for the specificities of a given problem.

Another trend is to make use not only of more data, but also of new types of data. Many types of data are nowadays passively collected and are largely unexploited, such as those provided by social networks, scanner data, credit card records, web search queries, electronic medical records, insurance claim data, etc. They could complement or even be a substitute for more traditional and actively collected data. The mining of language data, such as online customer reviews, is also a challenge and can be used for so-called 'sentiment analysis' (see e.g., Pang et al. (2002)).

Returning to the issue of causality discussed in the previous section, it should be noted that prediction algorithms also provide new ways to test theories. Indeed, one can see how well one can predict the output Y with all variables but X and/or how much the inclusion of a given variable (or a group of variables) X helps improving the prediction. One should be cautious, however, in drawing conclusions: the fact that a variable is not among the best predictors does not necessarily mean that it is not 'important'. For example, when Varian (2014) discusses differences in race in mortgage applications, saying that race is not among the best predictors is not the same as saying that evidence of discrimination does not exist.

In addition, the completeness of a given theory could be tested by confronting its predictions abilities against an atheoretical benchmark provided by machine learning.

14.5 Computational Issues

The collection and analysis of bigger and bigger data sets pose not only methodological challenges but obviously also computational challenges. Nevertheless, since there has been in the same time a tremendous increase in computing capabilities, researchers can handle larger and larger data sets using standard software and desktop computers. For example, maximum-likelihood estimation of dynamic factor models could be performed only with a small set of variables up to the late 90s (Stock and Watson (1989)), while recent research has shown how these models can be easily estimated in a high-dimensional context (Doz et al. (2012); Jungbacker and Koopman (2015)). In parallel with this increase in computing power, significant progress has been made in the development of fast and reliable numerical algorithms which scale well with the dimension. In particular, considerable research effort has been dedicated to improving the speed and performance of algorithms for Lasso regression.

In many situations, however, computational capability still represents an important constraint on our ability to handle and analyse big data sets. Methods that can handle thousands of variables may become inappropriate when moving to millions of variables. Moreover, some procedures can be particularly demanding in terms of computational complexity when applied to more than a handful of data. This is the case *e.g.*, for complex latent variable models for which closed-form solutions are not available. In this context there is a demand for extra computing power. Unfortunately the growth rate in computational capability of integrated circuits (CPU microchips) seems to be slowing down. However, thanks to technological progress driven by the video-game industry, new and fast growing computational power is coming from so-called graphics processing units (GPU) which allow for parallel computation and are easy to program. The general idea is that often it is possible to divide large problems into smaller independent tasks, which are then carried out simultaneously.

Splitting large problems into small ones is particularly natural in simulation-based Bayesian methods, which have recently attracted growing interest (see *e.g.*, Hoogerheide et al. (2009); Lee et al. (2010); Durham and Geweke (2013)). In Bayesian methods, the reduction in dimensionality is made by assuming prior distributions for the unknown parameters to infer and, whereas the computation of the so-called MAP (Maximum a Posteriori) estimator requires solving an optimisation problem, the computation of conditional means and covariances only requires integration, but in a high-dimensional space. For this task, one uses stochastic simulation methods and artificially generated random variables. Since the early days of Monte Carlo methods, there has been substantial development of new more sophisticated Sequential Monte Carlo and Particle Filter methods, allowing to deal with complex posterior distributions and more flexible econometric models.

Examples of successful applications of simulation-based Bayesian methods are reported by Billio et al. (2013a,b) and Casarin et al. (2013, 2015). The paper by Casarin et al. (2015) deals with the problem of conducting inference on latent timevarying weights used to combine a large set of predictive densities for 3712 individual stock prices, quoted in NYSE and NASDAQ, using 2034 daily observations from March 18, 2002 until December 31, 2009. The authors find substantial forecast and economic gains and also document the improvement in computation time achieved by using parallel computing compared to traditional sequential computation. Another application to nowcasting is discussed by Aastveit et al. (2014) who show that a combined density now-cast model works particularly well in a situation of early data releases with relatively large data uncertainty and model incompleteness. Empirical results, based on US real-time data of 120 leading indicators, indicate that combined density nowcasting gives more accurate density now-casts of US GDP growth than a model selection strategy and other combination strategies throughout the quarter with relatively large gains for the two first months of the quarter. The model also provides informative signals on model incompleteness

during recent recessions and, by focusing on the tails, delivers probabilities of negative growth, that provide good signals for calling recessions and ending economic slumps in real time.

14.6 Conclusions

Data are essential for research and policy. Definitely there is a trend towards empirical economics, and from this perspective, the advent of big data offers an extraordinary opportunity to take advantage of the availability of unprecedented amounts of data, as well as of new types of data, provided of course that there is easy access to them, in particular for academic research.

We have focused in this chapter on some methodological aspects of the analysis of large data sets. We have argued that many of the issues raised by big data are not entirely new and have their roots in ideas and work from the last decades. On the applied side, the applications with truly big data are still rare in economics although more research has been devoted in the last years to the use of relatively large but traditional data sets.

While in many problems the focus is shifting from identification towards prediction, which is a more '*revolutionary trend*', causality is still considered important and this duality is the matter for interesting debates in econometrics.

As concerns algorithmic and computational issues, the field of 'machine learning', a popular heading covering very different topics, is and will remain helpful in providing efficient methods for mining large data sets. However, one should be careful and not blindly import methodologies from other fields, since economic data structures have their own specificities and need appropriately designed research tools.

Undoubtedly, this research area calls for a lot of new, exciting and perhaps unexpected developments, within and outside the framework sketched here, and, if the data sets are big, the challenges ahead, in optimally exploiting the information they contain, are even bigger.

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